

Research on college mathematics teaching reform for emerging engineering education

Shuxia Wang

Department of Mathematics and Physics, Beijing Institute of Petrochemical Technology, Beijing, China.

E-mail: wangshuxia@bipt.edu.cn

Accepted 3rd July, 2020.

Abstract. The construction of emerging engineering education puts forward new requirements for talent training. College mathematics, as a core curriculum in the university's basic curriculum, plays a vital role in talent training for emerging engineering. However, the current mathematics teaching emphasizes theory but neglects practice. To solve this problem, this paper proposed a teaching method based on a combination of case-studying of emerging engineering applications and student-centered teaching paradigm reform. Furthermore, this paper gave a specific example of instructional design based on image processing.

Keywords: Emerging engineering education, college mathematics, case-studying, student-centered, instructional design.

INTRODUCTION

In order to meet the requirements of engineering talents for national strategy, industrial development and economic structural reform, the Ministry of Education of China started the construction of emerging engineering education in 2017. The goal of emerging engineering education is to cultivate innovative talents who can adapt to the new normal of economy with international competitiveness and strong engineering practice capabilities (Zhong, 2017). The introduction of this educational philosophy reflects that Chinese higher engineering education should focus on improving the practicality and innovation of talent training. From the perspective of economic development strategy, it requires higher education to strengthen cooperation with the industry, focusing on student engineering literacy and cultivation of practical ability. Therefore, the practical education and teaching system of colleges and universities is the key for training of emerging engineering talents (Chen and Chen, 2017). To meet the requirements of the cultivation of emerging engineering talents, it is necessary to continuously improve students' learning interest and promote the integration of basic course

learning and technological innovation in the course of basic course teaching.

College mathematics, including calculus, linear algebra, and probability theory, plays an important role in the training of engineering talents. However, affected by traditional teaching mode, current mathematics teaching faces the following problems that are not conducive to the construction of emerging engineering education:

- Current teaching emphasizes theory but neglects practice. As everyone knows, the most significant feature of college mathematics is the abstractness of theory and wide applications. Due to the abstractness of theory and massive content with few course hours, teachers often focus more on the rigor of logical reasoning and the integrity of the knowledge system, while ignoring the applications in practice. Few cases studying are used.
- The design of the case-studying is unreasonable. On the one hand, some teaching cases have been passed down from generation to generation, and the application scenarios are not related to students' living environment. This is difficult to stimulate students' learning interest. On

the other hand, although there are some cases related to modern technologies in which students are also interested, due to students' limited knowledge, teachers can only tell them that these problems can be solved without telling them how to solve. This makes the students doubt about the practical applications, which in turn discourages them from learning.

- Low student participation. Under the current teaching mode, students passively receive abstract theoretical knowledge and simple case introductions from teachers. They have only a limited understanding of limited applications. In the process of solving complex problems, it is difficult for them to participate deeply. Therefore, they are difficult to appreciate the happiness brought by problem solving, and then lack initiative and participation in learning.

The above problems have greatly hindered the development of students' innovative ability and practical ability. The current teaching mode is difficult to meet the requirements for talent training in the construction of emerging engineering education. How to change this situation is an urgent problem for the reform of college mathematics teaching.

COMBINATION OF CASE-STUDING AND STUDENT-CENTERED TEACHING

To solve the above problems, this paper proposes a teaching method based on combination of case-studying of emerging engineering applications and student-centered teaching paradigm reform (Cornejo *et al.*, 2018; Zhao, 2016).

Case-studying of emerging engineering applications

To design intuitive and interesting application cases for emerging engineering education, the mathematics thinking should be throughout in the teaching process, regarding the innovative ability and practical ability as the primary goal of teaching (Li, 2017). When designing the cases, we adhere to the design idea of "proposing problems—introducing concepts—exploring knowledge—solving problems—expanding applications".

The goal of case-studying is twofold. On the one hand, it enables students to experience the application of mathematical knowledge in practice, which can arouse their interest in exploration. On the other hand, it should develop and exercise their ability to analyze and solve problems.

Student-centered teaching paradigm

To further arouse the interest of students and improve the

quality of teaching, this paper proposes to reform from the traditional teacher-centered teaching paradigm to student-centered teaching paradigm. The student-centered teaching reform has three central concerns: student development, student learning and learning outcomes. Corresponding to these three concerns, the teaching process is divided into three steps. First, we determine the teaching goals of a course according to the graduation requirements, and then determine the teaching goals of each lesson. Secondly, to help students to learn theoretical knowledge by themselves, teachers should provide learning outlines and studying resources (recorded course or online course link) for students before each lesson. Finally, during the class, teachers should explain the difficulties in the self-study process and guide students to solve practical problems. In this teaching process, students become the center of learning activities. The role of the teacher changes from a simple knowledge transmitter to an organizer, coordinator, and mentor (Keiler, 2018).

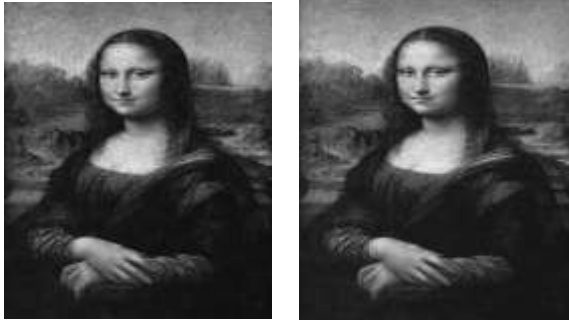
The student-centered teaching reform has rigorous scientific foundations: brain science and neurosciences, studies of emerging adulthood study and undergraduate development, cognitive sciences, learning psychology and learning science (Zhao 2017a,b). All of these foundations guarantees the effectiveness of the reform, and this reform has been practiced in many fields (Kulkarni and Vartak, 2019; Newton and Feinberg, 2020; Vinu and Sherimon, 2019).

This reform of the teaching paradigm can not only greatly arouse students' enthusiasm for learning and improve learning participation, but also greatly reduce the class hours required for theoretical knowledge (Adusumilli, 2014; Sumtsova *et al.*, 2017). Therefore, it can solve the problem of insufficient class hours after case expansion.

DESIGN

Taking eigenvalues and eigenvectors of matrices as an example, we present instructional design for the course of linear algebra. The eigenvalues and eigenvectors of matrices are widely used in emerging engineering. However, it is difficult to display them in an intuitive and easy-to-understand manner. To solve this problem, we carefully select a common application in our life, image compression, to introduce related concepts.

The case of image compression is intuitive and easy to understand. It can be used throughout the course of linear algebra. We can use the representation of images to introduce the basic concept of matrices. The dimensions of an image correspond to the dimensions of a matrix. A monochrome image corresponds to a common matrix and a color image corresponds to a three-dimensional matrix. The translation, scaling, and symmetry of an image can be realized by matrix multiplication. The concept of maximum



a. Original image b. Compressed image

Figure 1. Images before (a) and after (b) compression.

irrelevant group can be introduced through the synthesis of color images. The red (R), green (G), and blue (B) on the computer palette constitute a maximum irrelevant group of color sets. Each color corresponds to a set of RGB values, which represents the coefficients when the color is linearly combined by the three primary colors of red, green, and blue. Image compression can be achieved by choosing the largest eigenvalues and corresponding eigenvectors of the corresponding matrix. Therefore, all the content of a course can be connected through one case. This is also conducive to students' overall grasp of the course content and a deeper understanding of the relationship between the contents of each part.

Case introduction

With the rapid development and wide applications of mobile devices, we are accustomed to recording our lives with mobile phones. As camera pixels continue to increase, the cost of storing and transmitting pictures (images) also increases. How to compress the pictures without losing the original picture information (clarity) becomes very important.

In Figure 1, the picture has about 12 million pixels. Storing the original picture needs about 12 million elements of storage space. For the compressed picture in Figure 1b, it requires only about 710 thousands elements of storage space. The required storage space of the compressed picture is less than 6% of the original one, but the clarity of the two pictures is almost the same. Students often have the question "How is an image compressed?" In fact, it only needs to extract part of the eigenvalues and eigenvectors of the matrix corresponding to the picture. So far, this example has aroused students' interest in exploring eigenvalues and eigenvectors.

Knowledge construction

Self-study of basic knowledge before class

Before the class, the teacher provides a preview outline.

The students should complete the following questions by watching the teaching videos provided by the teacher:

- (1) What are the definitions of eigenvalues and eigenvectors?
- (2) Can the eigenvector be a zero vector?
- (3) Summarize the method of calculating eigenvalues and eigenvectors of a matrix. Try to calculate the eigenvalues and eigenvectors of a matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ (Reference answer: the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ has two eigenvalues $\lambda_1 = 3, \lambda_2 = 1$, and the eigenvectors corresponding to them are $k_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $k_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ respectively, where $k_1, k_2 \neq 0$.)
- (4) Try to explain the geometric significance of eigenvalues and eigenvectors.

Strengthen the key knowledge in class

In class, the teacher summarizes the main content and explains the difficulties according to student's feedback from preview. In addition, the teacher emphasizes the key points in the concept and solution. For example, the teacher should explain the geometric meaning of the eigenvalues and eigenvectors. By the definition, if the following equation is true, the vector x is an eigenvector of the matrix A and the scaling ratio λ is the corresponding eigenvalue.

$$Ax = \lambda x$$

According to this equation, if the original vector x and the transformed vector Ax by matrix A are collinear, the vector x is an eigenvector of the matrix A .

To make students have a better understanding of the concepts, we can visualize the relation between a matrix and its eigenvectors by MATLAB. The MATLAB function eigshow() presents a graphical experiment showing the effect on the unit circle of the mapping induced by various

2-by-2 matrices. For matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, by MATLAB

command "A=[2 1; 1 2]; eigshow(A)", the locus of the unit vector x and corresponding Ax can be obtained, as shown in Figure 2. The former is the green circle and the latter is the blue ellipse. When x and Ax are collinear, x is the eigenvector of matrix A .

The eigenvectors in Figure 2a and Figure 2b are the unit eigenvectors $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $-\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, corresponding

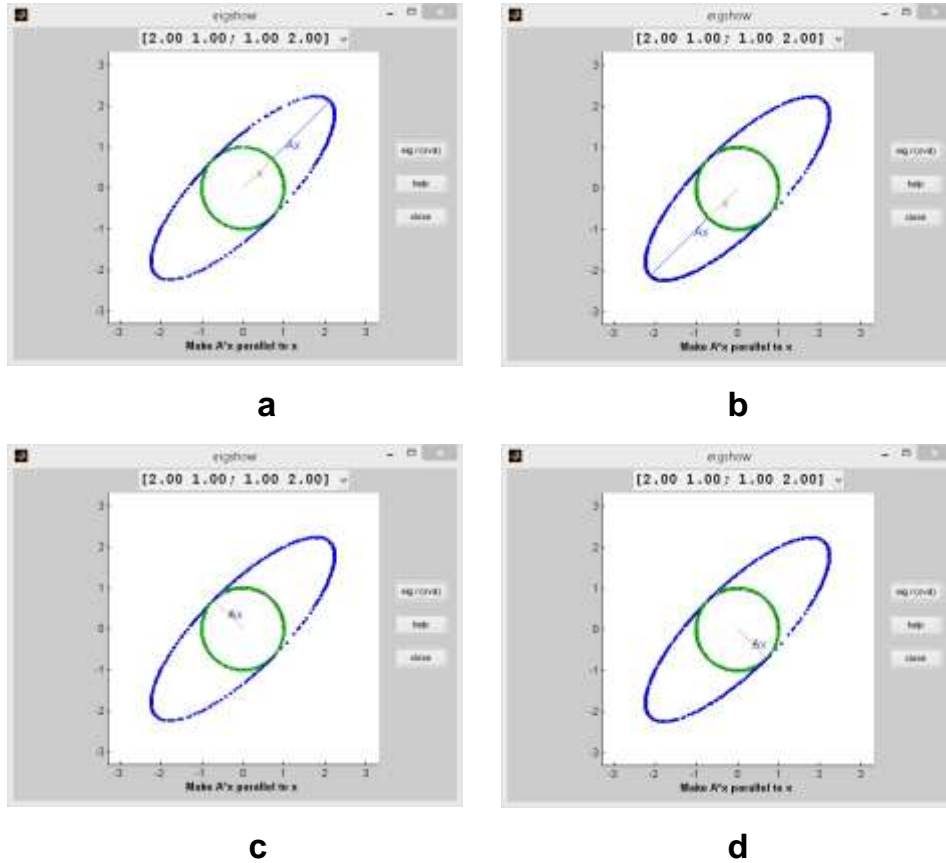


Figure 2. Graphical demonstration of eigenvalues and singular values by MATLAB function eigshow (A).

to the eigenvalue $\lambda_1 = 3$. The eigenvectors in Figure 2c and Figure 2d are the unit eigenvectors $\frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $-\frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, corresponding to the eigenvalue $\lambda_2 = 1$.

Applications

Eigenvalues and eigenvectors have important applications in many fields, such as stock market analysis, signal processing, image compression, and personalized recommendations. The main idea of these applications is to reduce the dimension of data. The most widely used methods are principal component analysis (PCA) and Singular Value Decomposition (SVD). In the following, we take SVD as an example to explain the idea of dimension reduction (Kalman, 1996).

Let matrix A denote the data, such as an image, to be processed. The dimension of matrix A is $m \times n$. First, matrix A can be decomposed into three small matrices:

$$A_{m \times n} = U_{m \times s} D_{s \times s} V_{s \times n}$$

Where D is a diagonal matrix with the square roots of the non-zero eigenvalues of AA^T or $A^T A$ on the main diagonal arranging in descending order, the column vectors of matrix U are the eigenvectors of AA^T and the row vectors of matrix V are the eigenvectors of $A^T A$ corresponding to eigenvalues in matrix D .

As shown in Figure 3, if we only take the three parts shown in red (part of the largest eigenvalues and their corresponding eigenvectors), their product is approximately equal to matrix A . Therefore, the reduction of data dimension is achieved.

Taking image compression introduced at the beginning of Section 3.1 as an example, we can compare the compression effect when retaining a different number of eigenvalues, as shown in Figure 4.

When the first 100 eigenvalues are retained, the sharpness of the compressed picture is almost the same as the original picture. In fact, matrix A be approximated as follows:

$$A_{4289 \times 2835} \approx U_{4289 \times 100} D_{100 \times 100} V_{100 \times 2835}$$

The storage space required for the right three small matrices is only 5.9% of that required for matrix A . Why

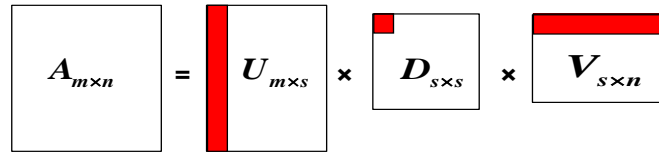
$$A_{m \times n} = U_{m \times s} \times D_{s \times s} \times V_{s \times n}$$


Figure 3. Illustration of SVD.



Figure 4. Compression effects of retaining different number of eigenvalues.

Table 1. Questionnaire survey of the effects of case-studying based teaching.

Question	Former survey (%)	Latter survey (%)	Improvement (%)
1. Understanding the importance of linear algebra	68	84	16
2. Understanding some important ideas and methods of linear algebra	49	79	30
3. Can accept new ideas, new theories and new methods	83	92	9
4. Learning linear algebra can exercise practical ability and creative ability	66	93	27
5. Linear algebra is important for the development of your profession	64	79	15
6. The learning of linear algebra can promote the learning of professional courses at the level of thinking	79	89	10
7. Can find the application of linear algebra in life	47	77	30

can we save so much information in the original picture with so little data? The reason is that the larger the eigenvalue, the more information it carries in the direction of its eigenvector. About this conclusion, we let it as a suspense here, and we will give a detailed proof after learning the knowledge of quadratic form.

Expanded thinking

After class, besides a moderate amount of exercise to ensure the learning effects of theoretical knowledge, extended thinking questions can also be given. For

example, the previous case showed the compression of monochrome pictures, so how should color pictures be compressed?

In fact, color pictures can be layered according to red, green, and blue, which can be compressed and then synthesized. This question can not only make students understanding the applications of eigenvalues and eigenvectors, but also understanding the significance of the largest unrelated group. In addition, the students need to study the MATLAB commands to compress color pictures themselves. Therefore, appropriate extended questions can investigate students' comprehensive use of new and old knowledge. In addition, they can cultivate

students' self-learning ability.

EMPIRICAL STUDY

In the fall semester of 2019, the proposed case-studying method based on emerging engineering was used for 98 students in the class. In order to evaluate the effects of the proposed method, a questionnaire survey was conducted on the students in class before and after the course. Five candidate answers were set for each question: A (strongly agree), B (agree), C (uncertain), D (disagree), and E (strongly disagree). The questionnaire survey included some questions related to the construction of emerging engineering education. The percentages of students who chose A or B in the former and latter survey were calculated as shown in Table 1.

It can be seen from Table 1 that the case-studying based method significantly improved the teaching effects. Of course, this method puts forward higher requirements for teachers while improving the teaching effects. How to design a practical and easy to understand application cases will be the direction we continue to work.

CONCLUSION

In this paper, we studied the college mathematics teaching reform for emerging engineering education. We proposed a teaching method based on combination of case-studying of emerging engineering applications and student-centered teaching paradigm reform. Specifically, an example of case-studying of matrix related concepts in linear algebra. An empirical study on a class of 98 students showed that the proposed method significantly improves the teaching effects.

APPENDIX-MATLAB CODE FOR IMAGE COMPRESSION

```
clear;close all;
%Load in image data
grayImage = imread('gray.jpg');
figure(1), subplot(141), imshow(grayImage, []);
title('original picture');
```

```
%Perform SVD decomposition
[U, D, V]=svd(im2double(grayImage));
```

```
%Rebuild and display the reconstructed image
ndim = 10; % Retain the first 10 eigenvalues
reconstructedImage(:, :, 1) =
U(:,1:ndim)*D(1:ndim,1:ndim)*(V(:, 1:ndim));
figure(1), subplot(142), imshow(reconstructedImage, []);
title('Retain the first 10 eigenvalues');
```

```
ndim = 50; % Retain the first 50 eigenvalues
```

```
reconstructedImage(:, :, 1) = U(:, 1:ndim)*D(1:ndim,
1:ndim)*(V(:, 1:ndim));
figure(1), subplot(143), imshow(reconstructedImage, []);
title(' Retain the first 50 eigenvalues');
```

```
ndim = 100; % Retain the first 100 eigenvalues
reconstructedImage(:, :, 1) = U(:, 1:ndim)*D(1:ndim,
1:ndim)*(V(:, 1:ndim));
figure(1), subplot(144), imshow(reconstructedImage, []);
title(' Retain the first 100 eigenvalues');
% Different compression results can be obtained by
modifying the number of eigenvalues
```

ACKNOWLEDGEMENT

This work was supported by Student-centered Teaching Paradigm Reform Project of Beijing Institute of Petrochemical Technology (No. ZDFSGG20190803) and Science and Technology Plan of Beijing Municipal Education Commission (No.KM201910017002).

REFERENCES

- Adusumilli S (2014)**. "Active learning strategies: an illustrative approach to bring out better learning outcomes from science, technology, engineering and mathematics (stem) students". *Int. J. Emer. Tech. Learn.* 9(9):21-25.
- Chen H, Chen M (2017)**. "On Thoughts and Explorations of Training Emerging Engineering Talents in Comprehensive Universities". *Res. High. Educ. Eng.* (2):19-23.
- Cornejo MG, O'Hara B, Tarazona-Vasquez F, Barrios F, Power M (2018)**. "Moray: Bridging an Ancient Culture of Innovation with Emerging Pedagogies in Engineering". *Int. J. Eng. Pedagogy*, 8(4):43-55.
- Keiler LS (2018)**. "Teachers' roles and identities in student-centered classrooms". *Int. J. STEM Educ.* 5:34.
- Kalman D (1996)**. "A Singularly Valuable Decomposition: The SVD of a Matrix". *Coll. Math. J.* (27):2-23.
- Li J (2017)**. "Research on the cultivation of creative thinking and practical abilities in the university mathematical teaching". *Agro Food Industry Hi-Tech*, (28):2430-2434.
- Kulkarni A, Vartak R (2019)**. "A module integrating conventional teaching and student-centered approach for critical reading of scientific literature". *Biochem. Mol. Biol. Educ.* 47(5):581-588.
- Newton L, Feinberg DE (2020)**. "Assisting, Instructing, Assessing: 21st Century Student Centered Librarianship". *The Ref. Lib.* 61(1):25-41.
- Sumtsova OV, Aikina TY, Bolsunovskaya LM, Phillips C, Zubkova OM, Mitchell PJ (2017)**. "Collaborative learning at engineering universities: benefits and challenges". *Int. J. Emer. Tech. Learn.* 13(01):160-177.
- Vinu S, Sherimon PC (2019)**. "Staff Perceptions on Implementation and Challenges of Student Centered Pedagogy: A Case Study". *J. Res. High. Educ.* 3(1):37-60.
- Zhao JM (2016)**. "On the new three-centered theory: concepts & history". *Res. High. Educ. Eng.* (03):35-56.
- Zhao JM (2017a)**. "Open the black box: the scientific foundation of undergraduate learning and development (part 1) - a series of study of the SC undergraduate education reform in the USA (2)". *Res. High. Educ. Eng.* (03):31-52.
- Zhao JM (2017b)**. "Open the black box: the scientific foundation of undergraduate learning and development (part 2) - a series of study of the SC undergraduate education reform in the USA (2)". *Res. High. Educ. Eng.* (04):30-46.
- Zhong DH (2017)**. "Connotations and Actions for Establishing the Emerging Engineering Education". *Res. High. Educ. Eng.* (3):1-6.